Symbolic Initialization of Simultaneously Under- and Over-determined Models

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Outline

- Brief introduction of
  - Under-determined systems
  - Over-determined systems
- Similarity of under- and overdetermination
- Simultaneously under- and over-determined systems
- Conclusions

Under-determined systems

- Not enough initial conditions to fully specify the system
- Additional initial conditions for states need to be determined

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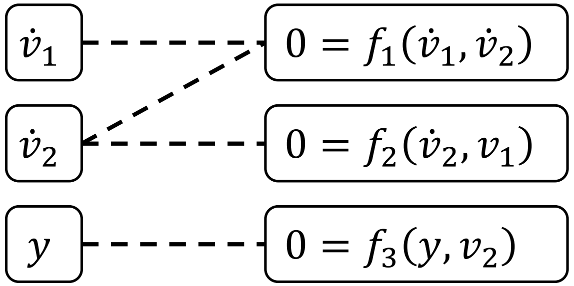
\begin{aligned}
0 &= f_1(\dot{v}_1, \dot{v}_2) \\
0 &= f_2(\dot{v}_2, v_1) \\
0 &= f_3(y, v_2)
\end{aligned}
$$



Reference: L. Ochel, B. Bachmann, "Initialization of Equation-Based Hybrid Models within OpenModelica", In EOOLT 2013 Proceedings, Nottingham, UK, April 19, 2013, pp. 97-103

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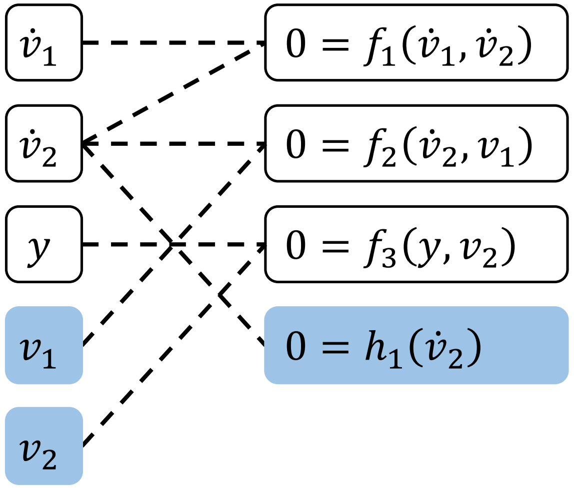

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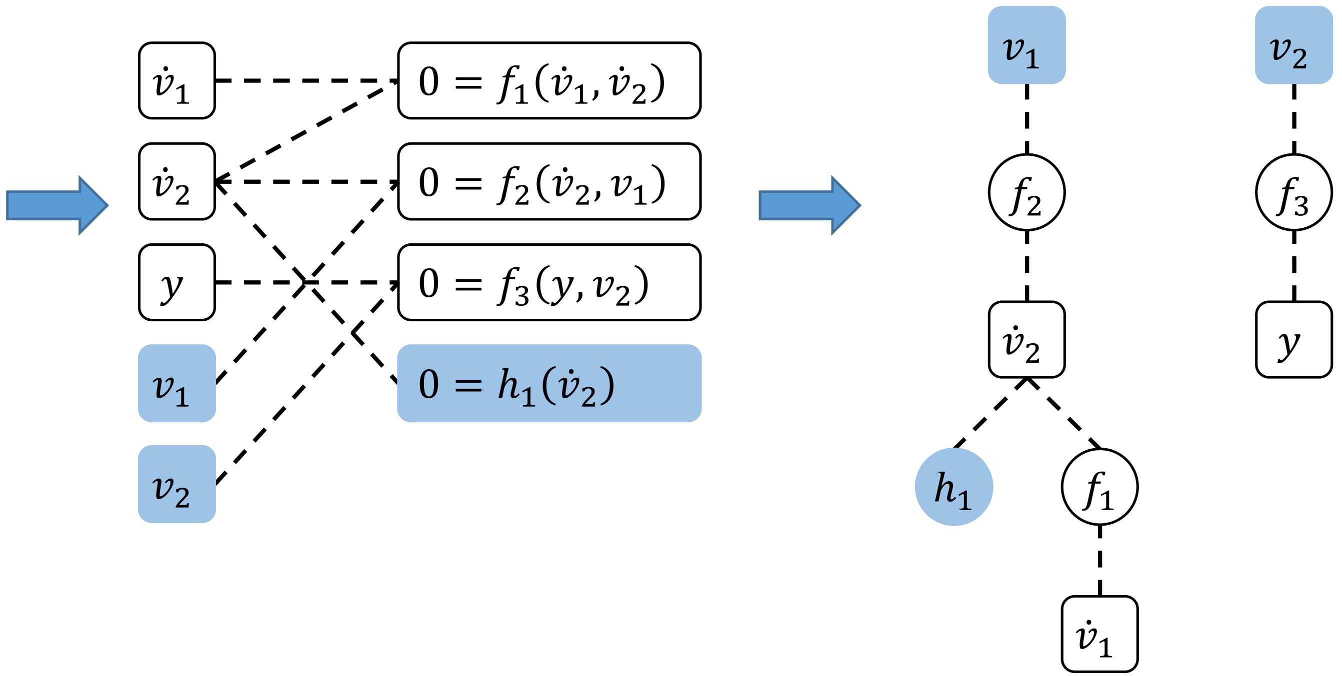

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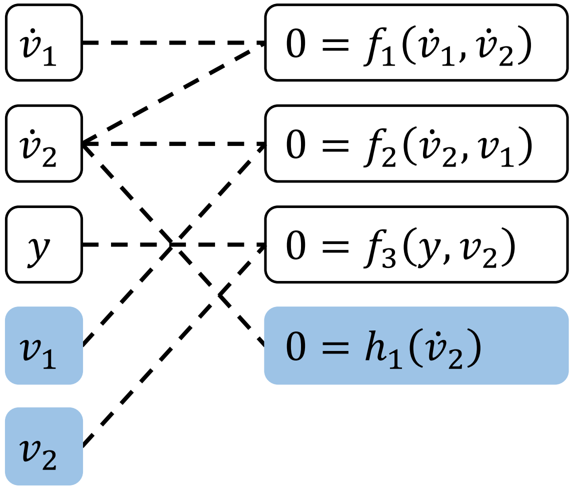

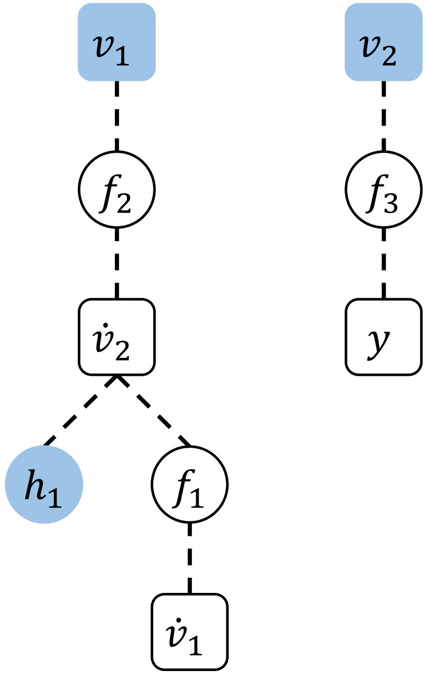

$\omega = (v_1 \quad v_2)$ contains additional unknowns, compared to the system for simulation

$$\frac{\partial \underline{h}}{\partial \underline{\omega}} = (* \quad 0)$$

0-colums of $\frac{\partial \underline{h}}{\partial \underline{\omega}}$ correspond with additionally needed initial conditions for corresponding ω_i

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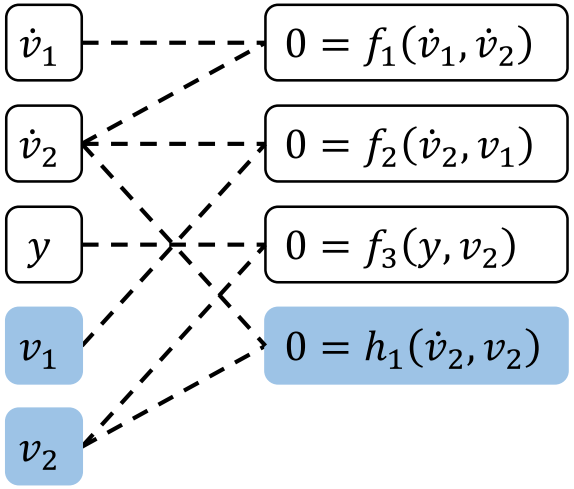

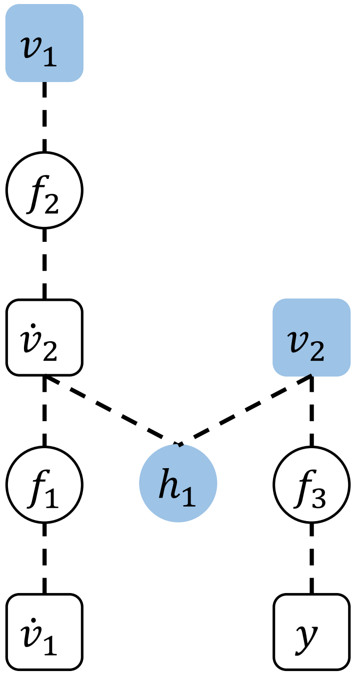

$\omega = (v_1 \quad v_2)$ contains additional unknowns, compared to the system for simulation

$$\frac{\partial \underline{h}}{\partial \underline{\omega}} = (* \quad *)$$

If there are not enough 0-colums to fulfil initial conditions, then randomly select as many ω_i as needed.

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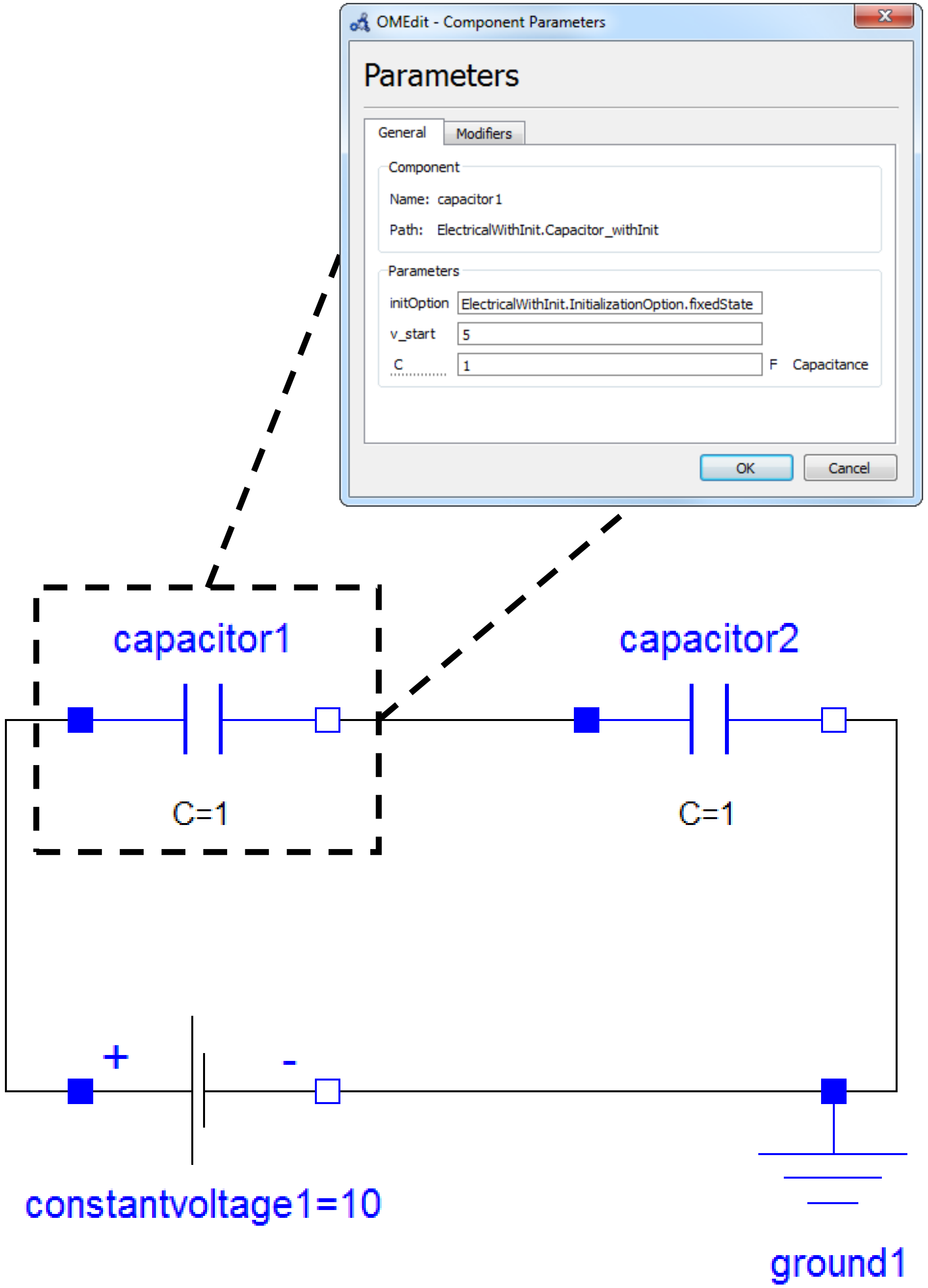
Over-determined systems

- Component-based models
- Initialization is part of components
- Over-determined problems arise by adding connections



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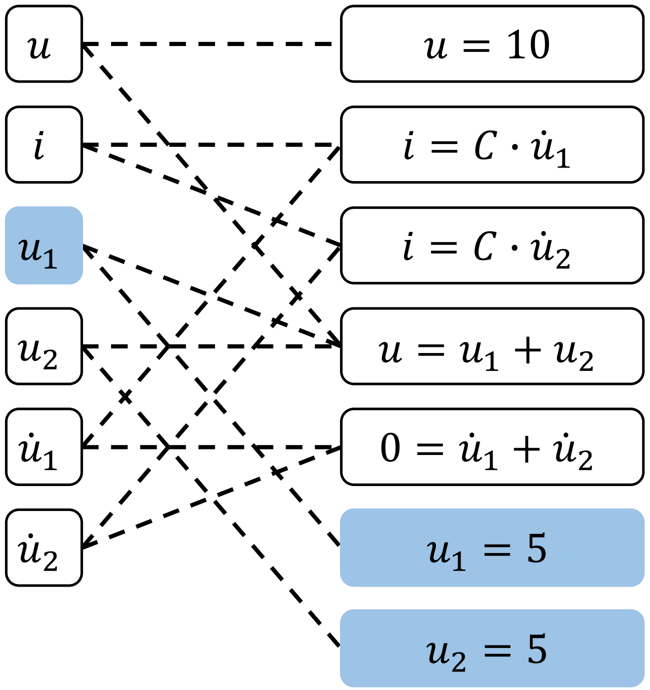

- Rather complex algorithm based on partial matchings

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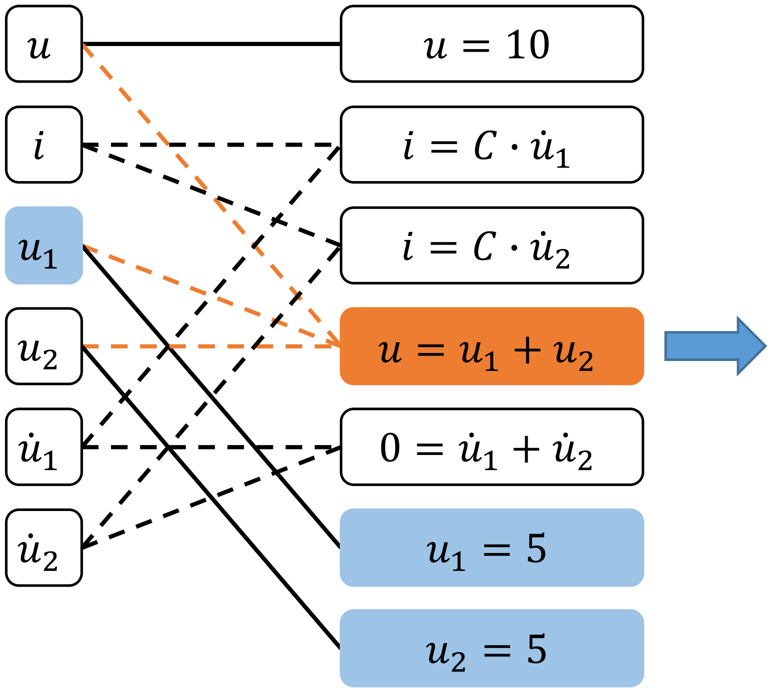
Over-determined systems

Find redundant equation



becomes unassignable and therefore get removed

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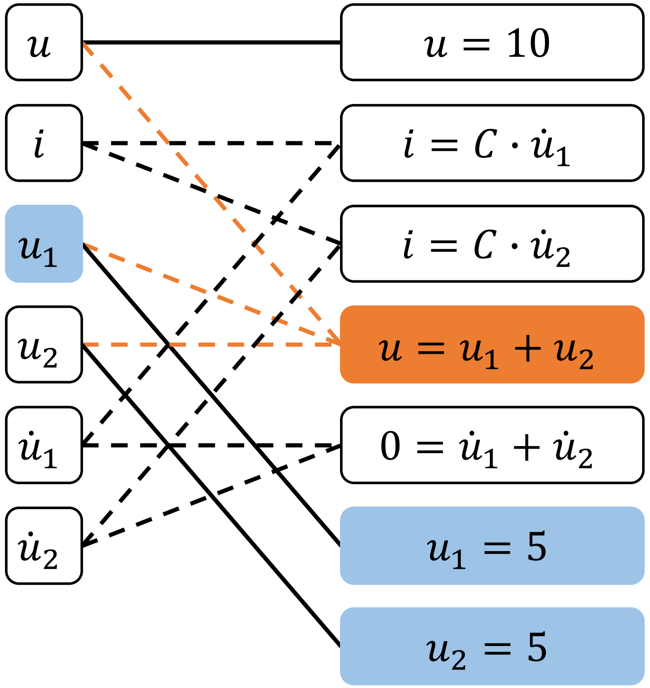

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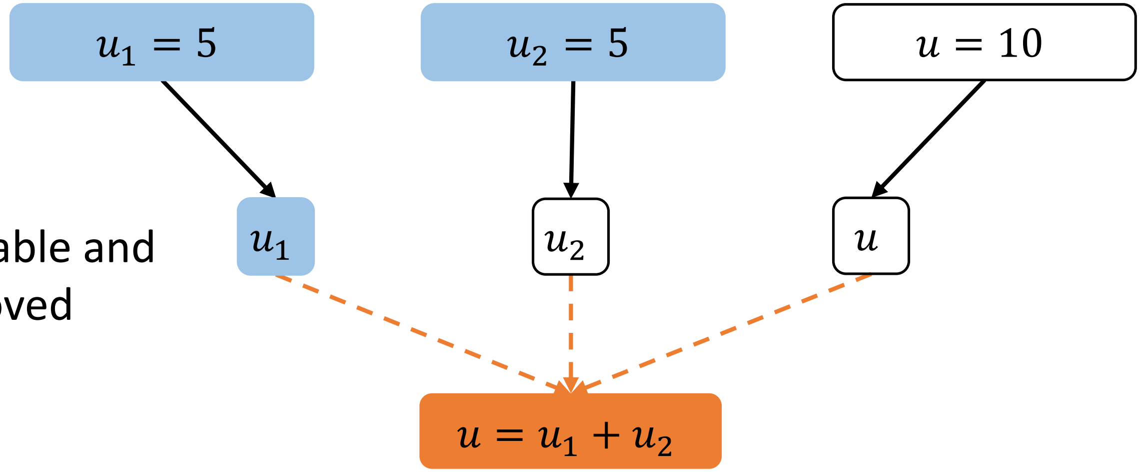
Verify if the removed equation is consistent



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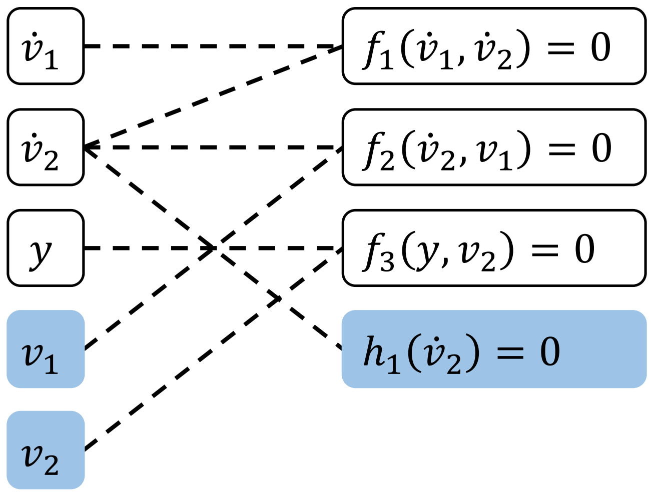
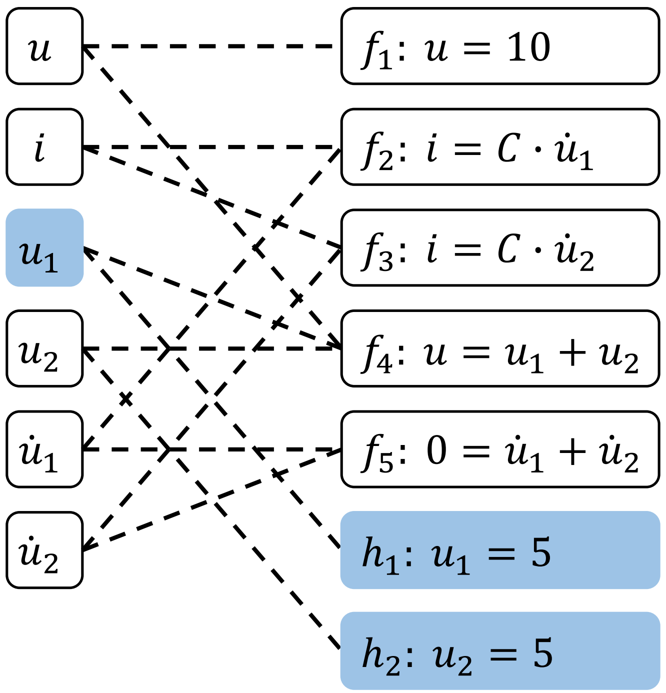
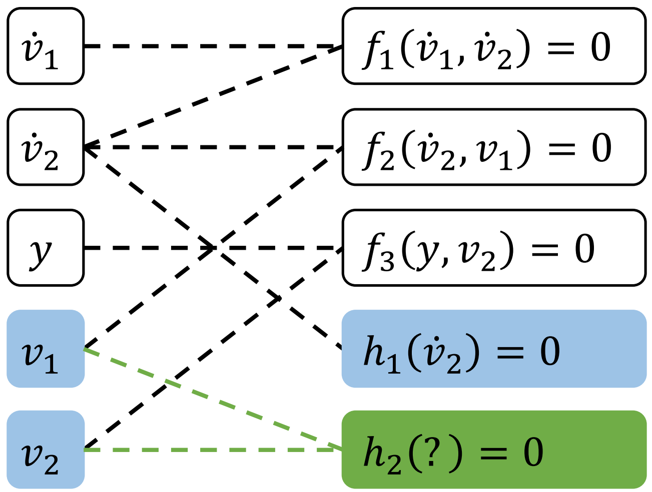
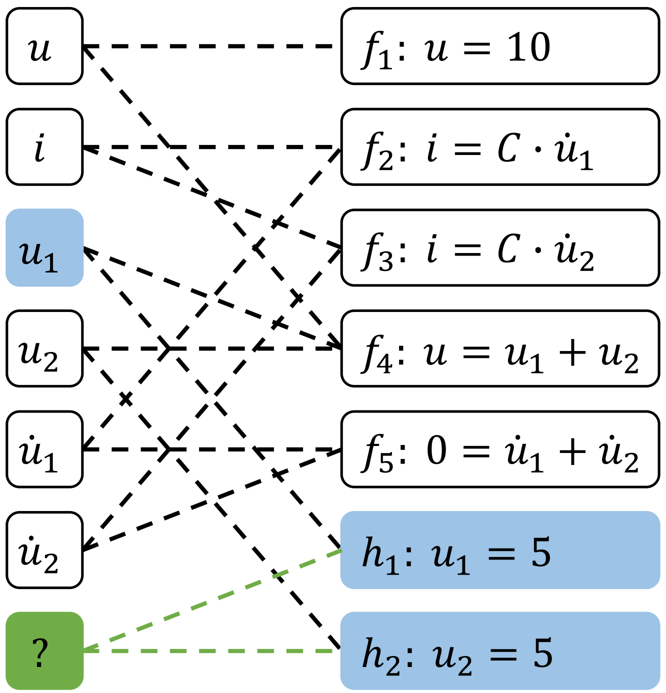
Similarity of under- and overdetermination



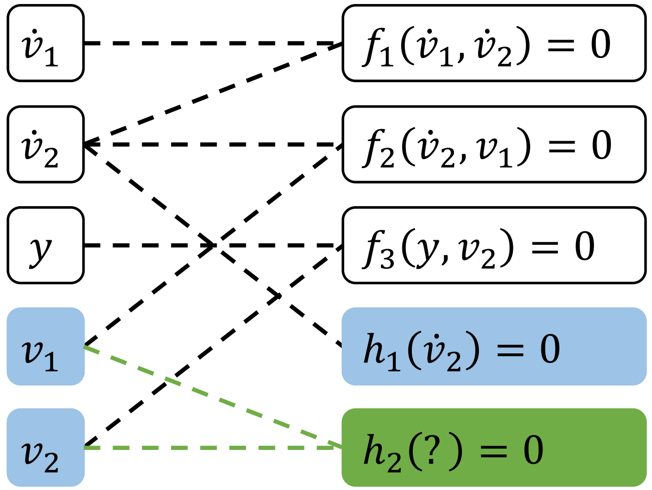


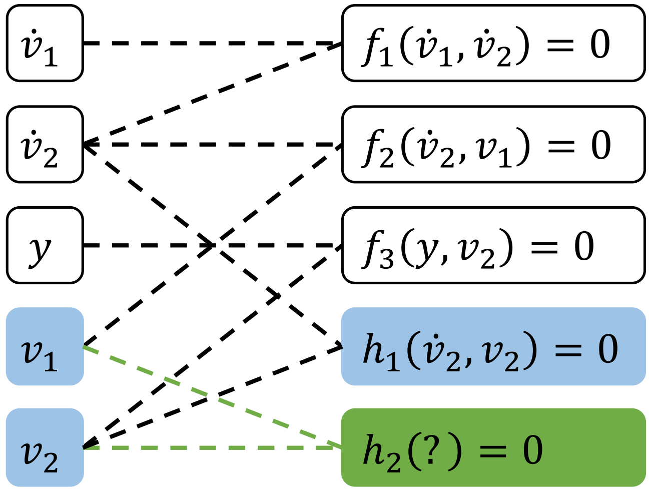
Similarity of under- and overdetermination





Fulfil under-determined systems





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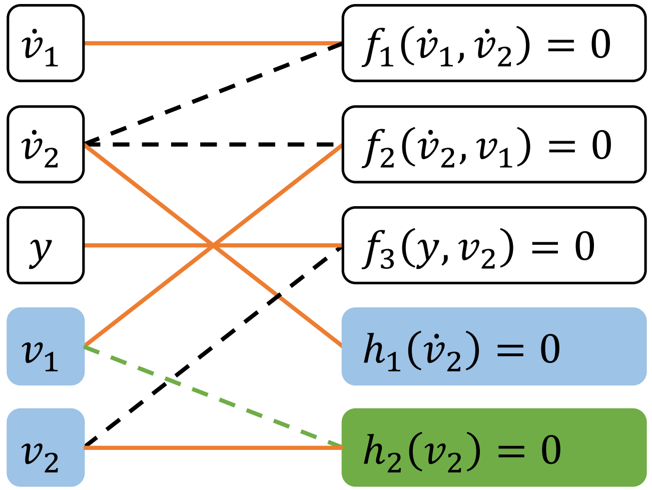
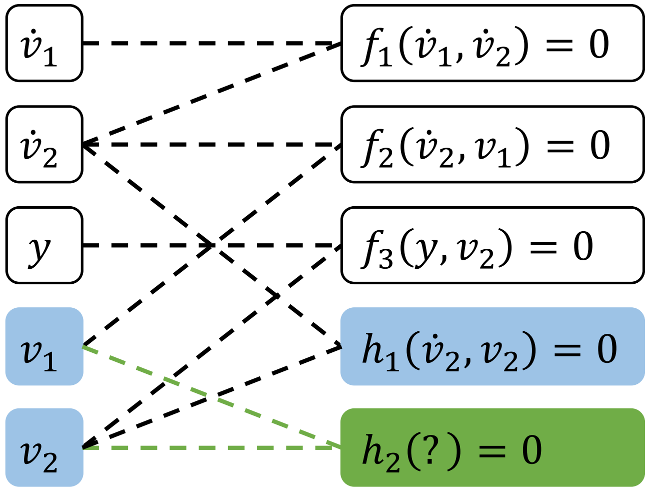
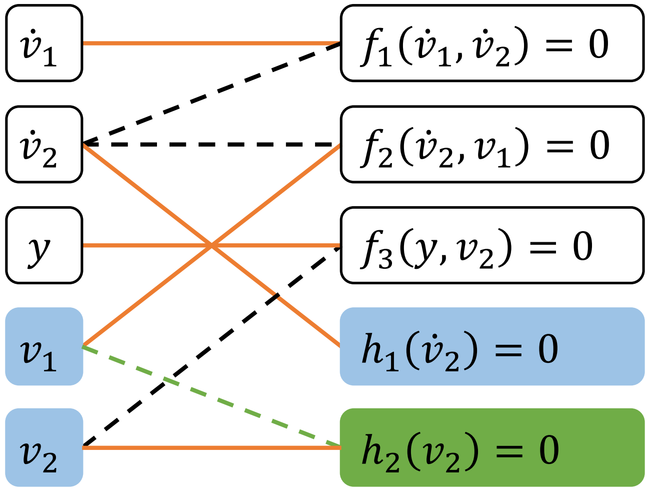




SCC1	h1
SCC2	f2
SCC3	h2
SCC4	f1
SCC5	f3

$$\frac{\partial h_1}{\partial \underline{\omega}} = \begin{pmatrix} * & 0 \end{pmatrix}$$

Fulfil under-determined systems



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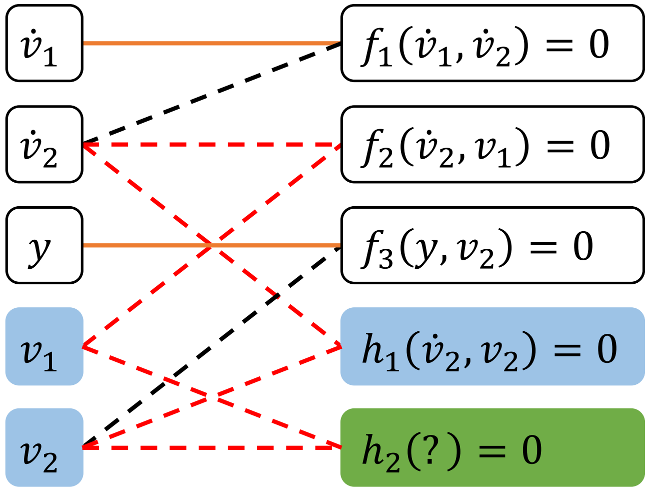


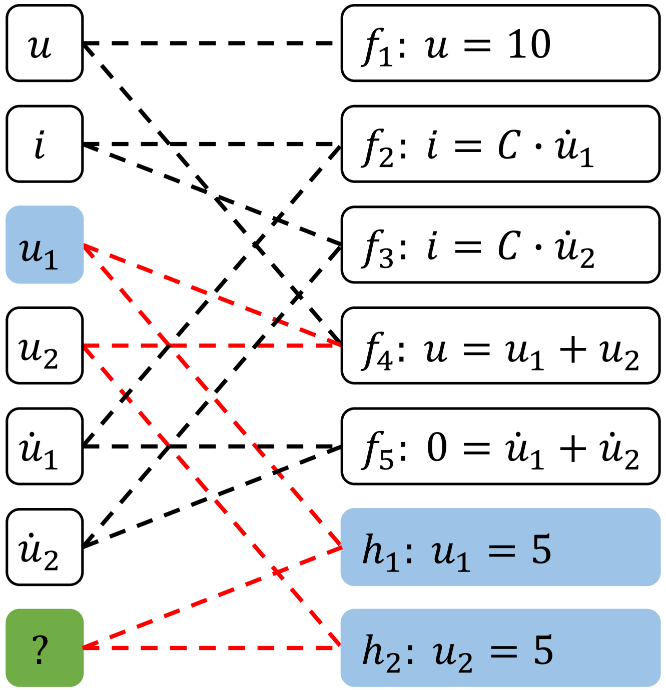
SCC1	f2, h1, h2
SCC2	f1
SCC3	f3

$$\frac{\partial h_1}{\partial \underline{\omega}} = \begin{pmatrix} * & * \end{pmatrix}$$

- Solvability of involved equations can be used to select proper initial conditions

Fulfil over-determined systems

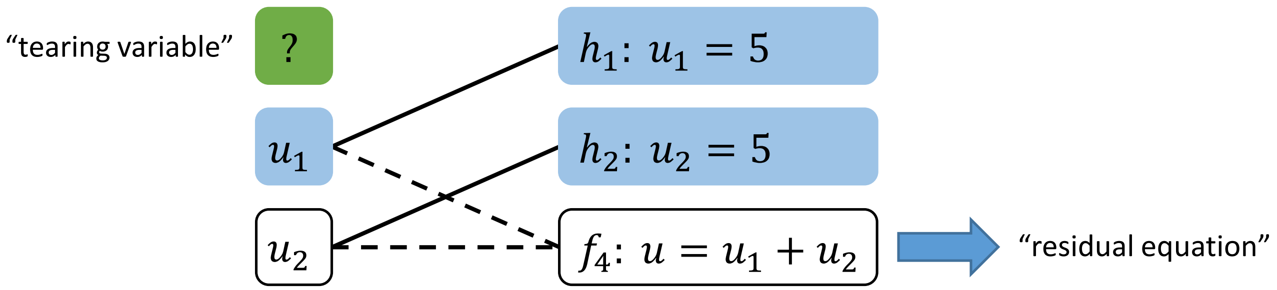
- ? will always be part of an algebraic loop

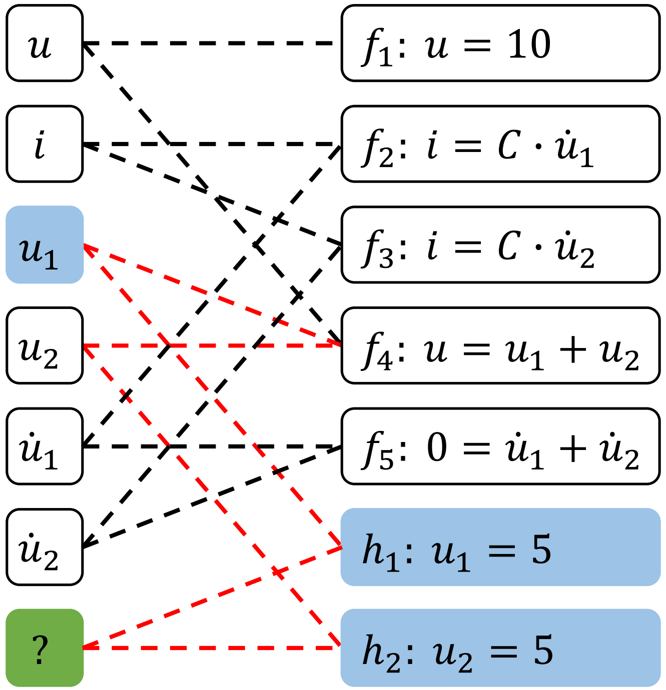


SCC1	f1
SCC2	f2, f3, f5
SCC3	h1, h2, f4

Fulfil over-determined systems

- ? will always be part of an algebraic loop
- applying tearing-alike method to select "redundant equations"





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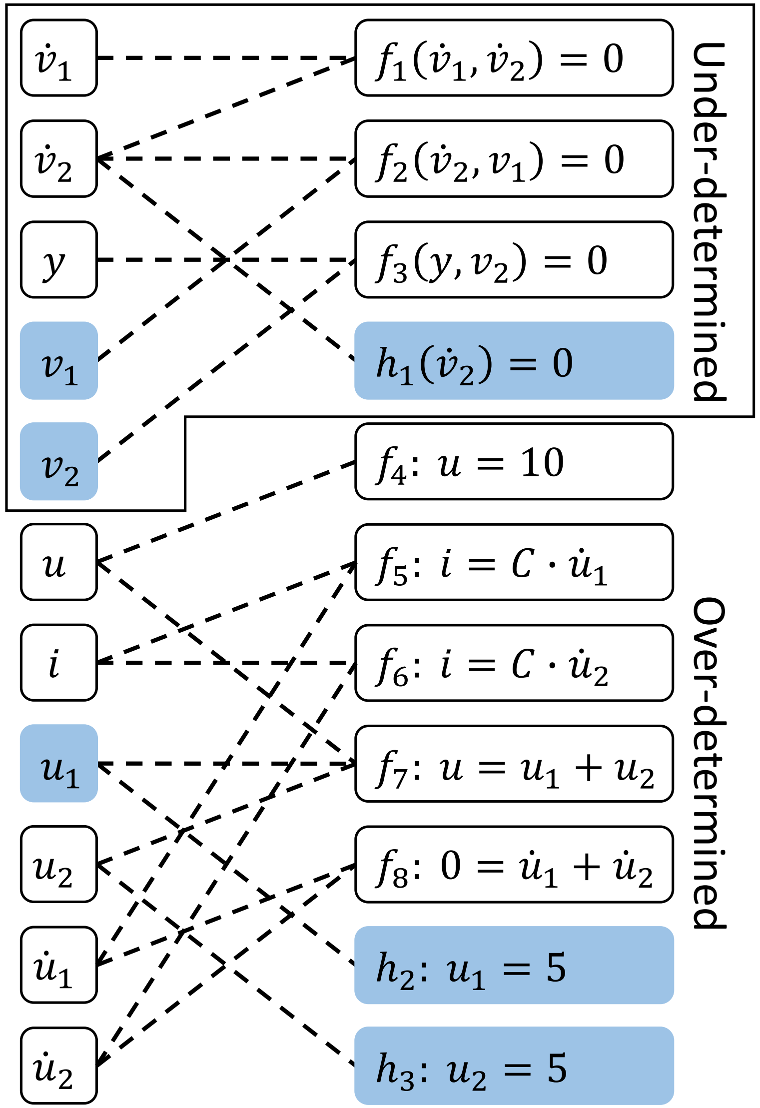
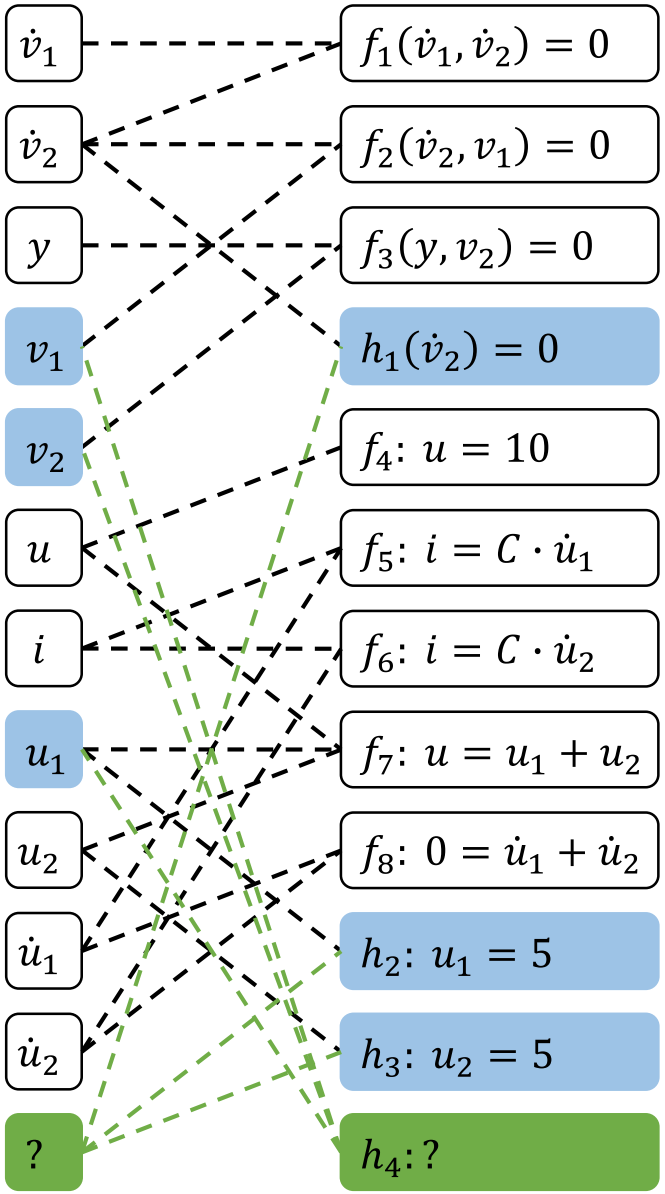
Simultaneously under- and overdetermined systems

- Those problems are always structural singular



Simultaneously under- and overdetermined systems

- Those problems are always structural singular
- Introducing one “green” variable node and equation node respectively
  - (multiple times if needed)
- Same approach as discussed before



Conclusions

- Under- and over-determined systems can be seen as similar problems
- That allows it to solve also simultaneously under- and over-determined problems
- Available in OpenModelica 1.9.2beta

Recent and further developments

- Removing numeric initialization approach
- Separating initialization/simulation strictly to allow more specific optimization
- Proper homotopy support

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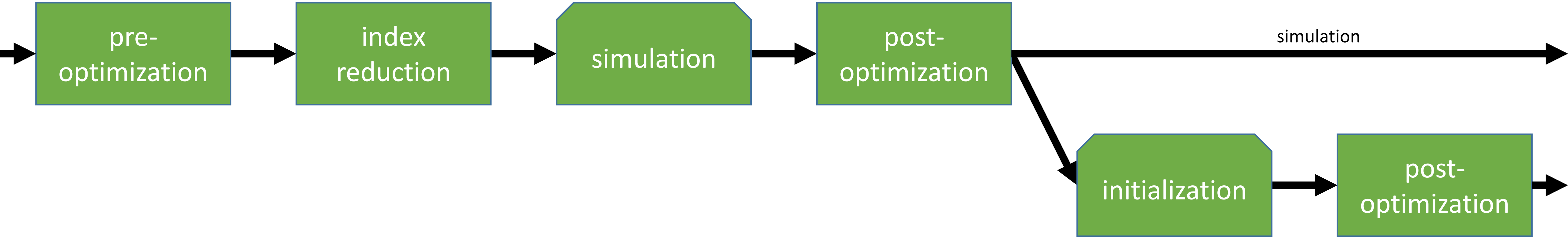

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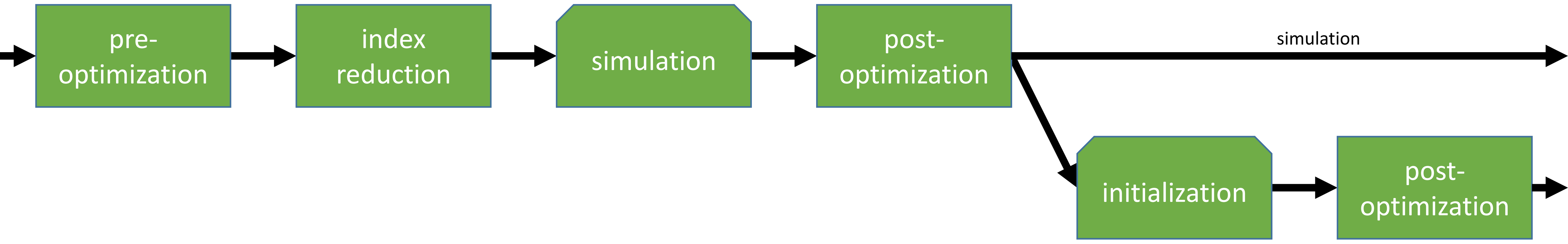

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