Automated Detection of Zeno Sets in Models by an OpenModelica Addon

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Outline

Bouncing Ball Model Falsified by Zenoness

Zenoness = infinite transitions in a bounded and finite length of time

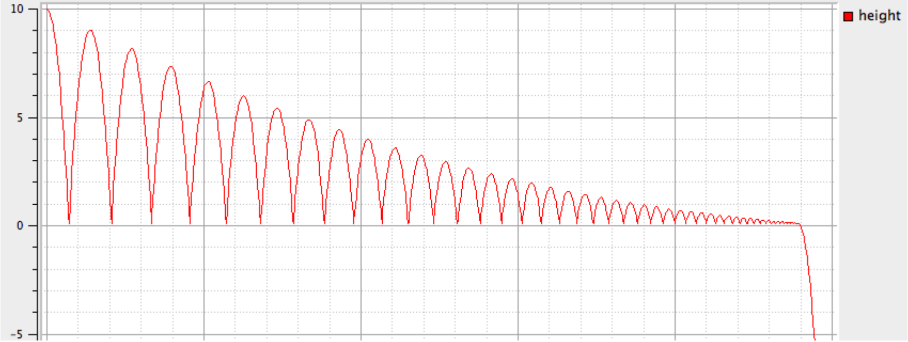

Figure: Bouncing Ball Simulation in OpenModelica

- Zenoness falsifies the result of the simulation.
- The ball drops below the surface.

Water Tank

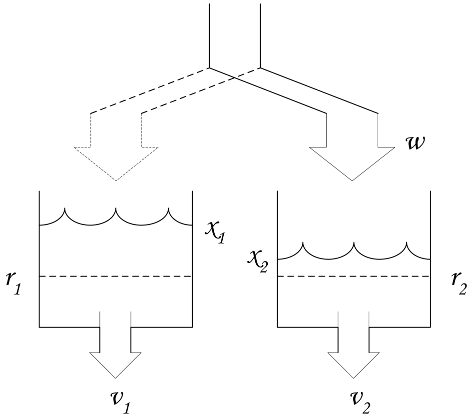

Figure: Water Tank System

- w, one water source
- v_i, hole in tank i, from which water drains
- x_i, current water level in tank i
- r_i, minimal required water level in tank i

$w < v_1 + v_2$ is the interesting case

Hybrid Automaton

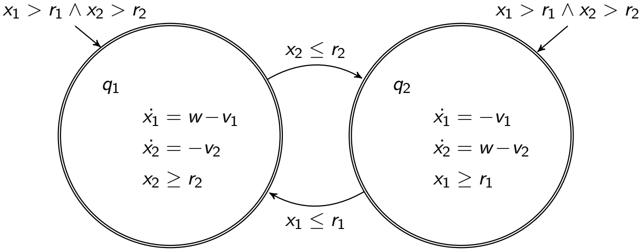

Figure: A Hybrid Automaton of the Water Tank System

- Q, discrete states: $\{q_1, .., q_n\}$
- X, continuous variables from $\mathbb{R}^n$ with $n \geq 1$
- *Init*, initial states from Q
- D: $Q \to \mathcal{P}(X)$, domain for each discrete state
- $E \subset Q \times Q$, transition from one state to another: $q_i \to q_j$
- Guards $G : E \to \mathcal{P}(X)$
- Reset map $R : E \times X \to \mathcal{P}(X)$
- $D(q)^0$, interior of the domain
- $\partial D(q)$, boundary of the domain

Requirements

A cycle is a necessary condition for a hybrid automaton to accept zeno executions [3].

Theorem (*Non-expanding* reset map ($|R| \leq 1$), zeno [3])

- *If* $G(q, q') \cap D(q)^0 = \emptyset, \forall (q, q') \in E$ *with* $q, q' \in Q_\infty$
- *then* $x_i \in \partial D(q_i)$ *for all* $i = 1, ..., m$.

Corollary (*Identity* reset map ($R = 1$), zeno free [3])

- $G(q, q') \cap D(q)^0 = \emptyset, \forall (q, q') \in E$,
- *for all cycles* $\{q_i\}_{i=1}^{K}$ *with* $q_K = q_1$ *and* $(q_i, q_{i+1}) \in E$, $1 \leq i \leq K - 1, \cap_{i=1}^{K-1} \partial D(q_i) = \emptyset$.

Steps of the Tool

Input: The hybrid automaton of the system, provided by the user

- Parse input file based on a predefined grammar
- Detect cycles
  - Robert Tarjan "Enumeration of the Elementary Circuits of a Directed Graph"[2]
- Detect the zeno sets
- Return results

Input File

Input File of Water Tank Automaton

```
Automaton;
State, q1, True, x2 >= r2, x1 = w - v1, x2 = - v2;
State, q2, True, x1 >= r1, x1 = - v1, x2 = w - v2;
Transition, q1, q2, x2 <= r2, 1;
Transition, q2, q1, x1 <= r1, 1;
```

Water Tank

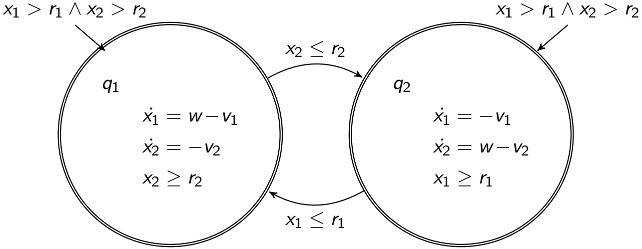

Figure: A Hybrid Automaton of the Water Tank System

Output of the Tool

```
*Zeno> main "watertank.txt"
The zeno set for the cycle [q1 q2 q1] is:
In state ``q1'' for the continuous variable x1 the zeno point(s) are: r1; for the
 continuous variable x2 the zeno point(s) are: r2;
In state ``q2'' for the continuous variable x1 the zeno point(s) are: r1; for the
 continuous variable x2 the zeno point(s) are: r2.
```

Validation of the Tool

Tested all combinations of guards and domains to validate that:

- Neither the order of the states and transitions nor the order in the restrictions matters.
- Only transitions of the cycles are considered.
- Intervals are correctly generated.
- Distinct cycles are correctly detected.
- Tool works correctly for either variables or constants as bounds.

State of the Tool

- Command-line prototype
- Written in Haskell

Next steps:

- Always handle reflexive edges correctly
- Automatic generation of hybrid automaton (use the HyAuLib?)
- Prevent zeno behavior
  - Introduce delay

Conclusion

- Zenoness is a modeling artifact.
- Prototype detects zeno sets off-line and automatically; requires transformed model.
- Zeno sets are important for validating the simulation.
- Validating the simulations of hybrid systems is crucial for validate hybrid systems.

Bibliography

[1] Marcel Gehrke. "Detection of Zeno Sets in Hybrid Systems to Validate Modelica Simulations". Bachelor thesis. Hamburg University of Technology, 2012.

[2] Robert E Tarjan. *Enumeration of the Elementary Circuits of a Directed Graph*. Tech. rep. Cornell University, Ithaca, NY, USA, 1972.

[3] Jun Zhang et al. "Zeno hybrid systems". In: *International Journal of Robust and Nonlinear Control* 11.5 (2001), pp. 435–451. ISSN: 1099-1239. DOI: `10.1002/rnc.592`. URL: `http://dx.doi.org/10.1002/rnc.592`.