## OpenModelica Annual Workshop 2012

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# Multiple Shooting with Collocation: An Efficient Approach to Dynamic Optimization, with Possibilities for Integration in OpenModelica

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# Motivation: NMPC using multiple shooting with collocation

NMPC is superior to traditional control in many cases

Critical issues:

- Accuracy
- Problem size - computational effort
- Safety specifications – path constraints

→ Fast algorithm is needed

# Nonlinear optimal control problem

$$\min_{u(t),x(t),y(t),p} \left\{ J = \Phi\left(x_f, y_f, t_f, p\right) + \int_{t_0}^{t_f} L\left(x(t), y(t), u(t), p\right) dt \right\}$$

$$\text{s.t.} \qquad \dot{x}(t) = f\left(x(t), y(t), u(t), t, p\right) \qquad x(t_0) = x_0$$

$$0 = g\left(x(t), y(t), u(t), t, p\right) \qquad t \in \left[t_0, t_f\right]$$

$$0 \le h\left(x(t), y(t), u(t), t, p\right)$$

$$0 = r_e\left(x(t_i), y(t_i), p\right)$$

$$0 \le r_i\left(x(t_i), y(t_i), p\right)$$

Bounds of state variables, controls and parameters

# Multiple shooting

**NMPC problem** → **problem transformation** → **problem solution**

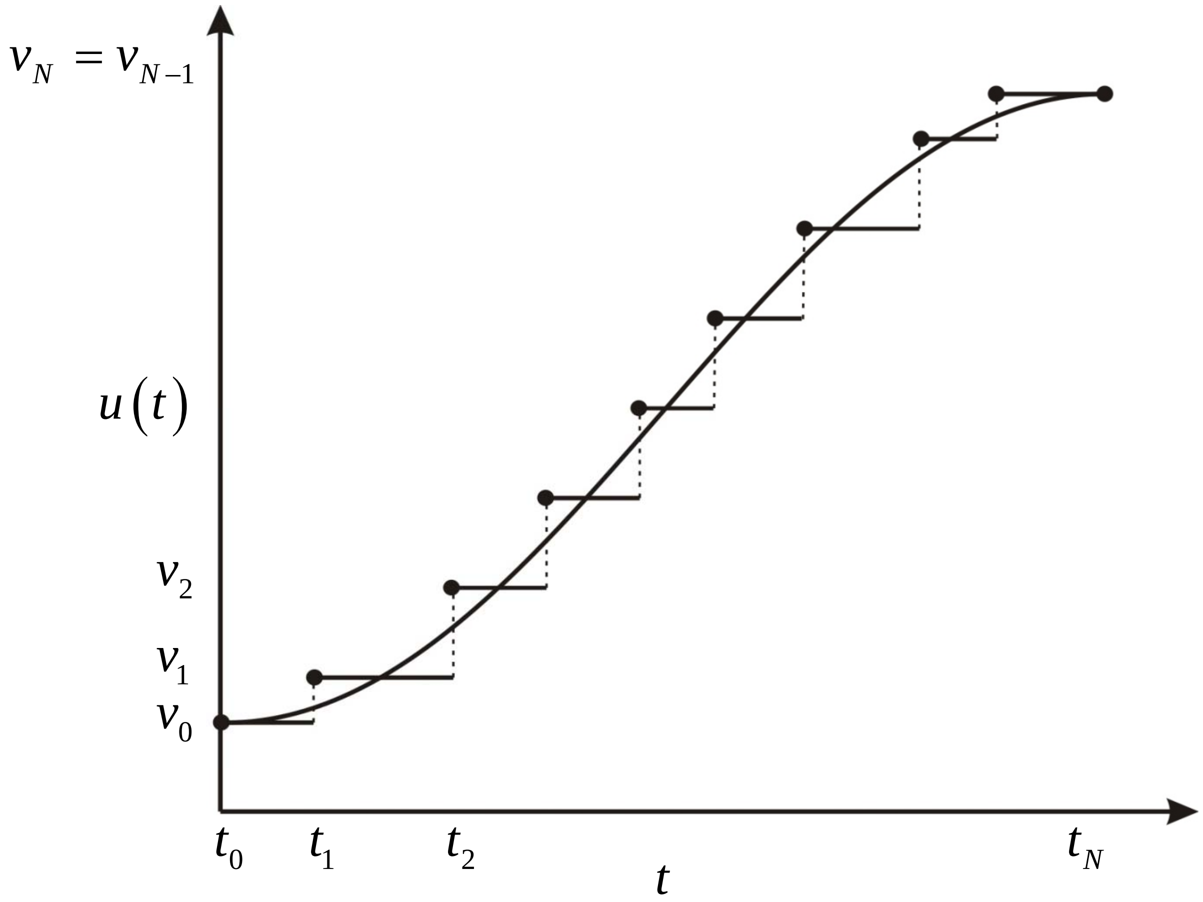
**problem transformation**

- discretization
- parametrization
- initial value problem (IVP)
- continuity equations

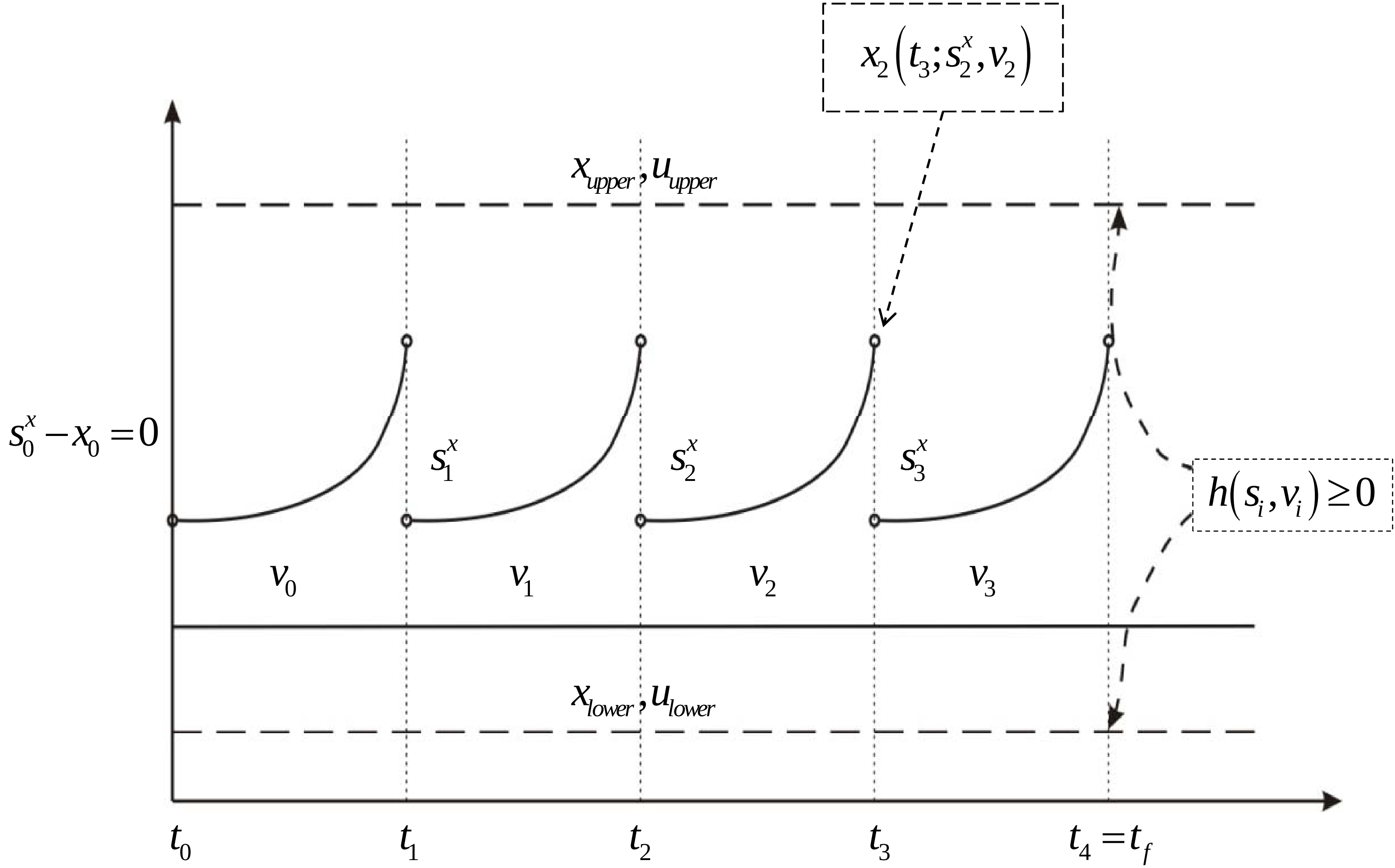
**problem solution**

- initial guess
- objective function
- simulation
- continuity
- path constraints

# Multiple shooting – control parametrization



# Multiple shooting – state parametrization



# Finite dimensional NLP problem

$$\min_{\substack{v_0,\dots,v_{N-1}\\ s_0,\dots,s_{N-1}}} \left\{ J = \Phi\left(s_N, p, t_N\right) + \sum_{i=0}^{N-1} L(s_i, v_i, p) \right\}$$

s.t.

$$0 = s_0 - x_0$$

$$0 = s_{i+1} - x_i\left(s_i, v_i, p, t_{i+1}\right) \qquad i = 0,1,\dots,N-1$$

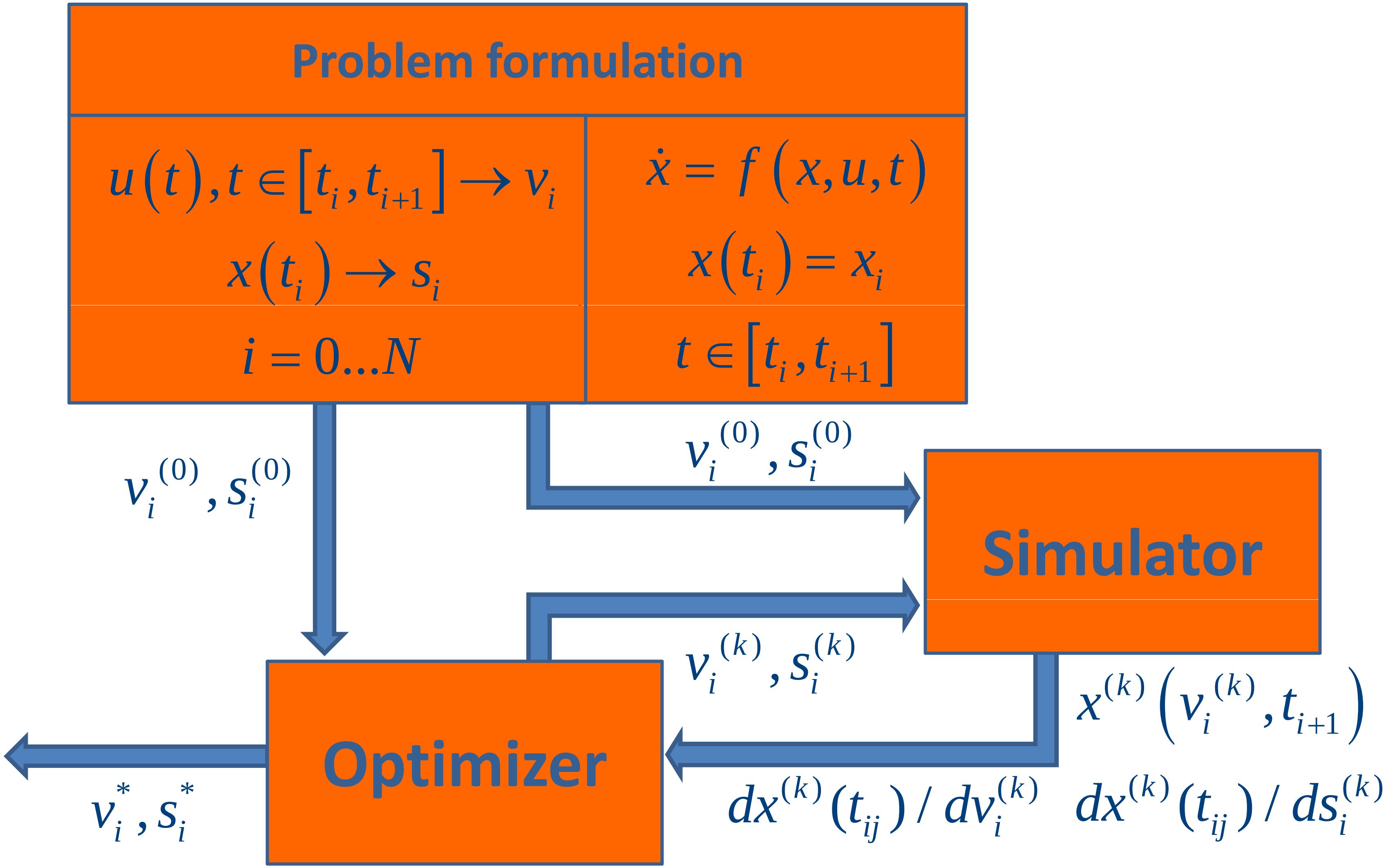
$$0 = g\left(s_i, v_i, p\right) \qquad i = 0,1,\dots,N$$

$$0 \le h\left(s_i, v_i, p\right)$$

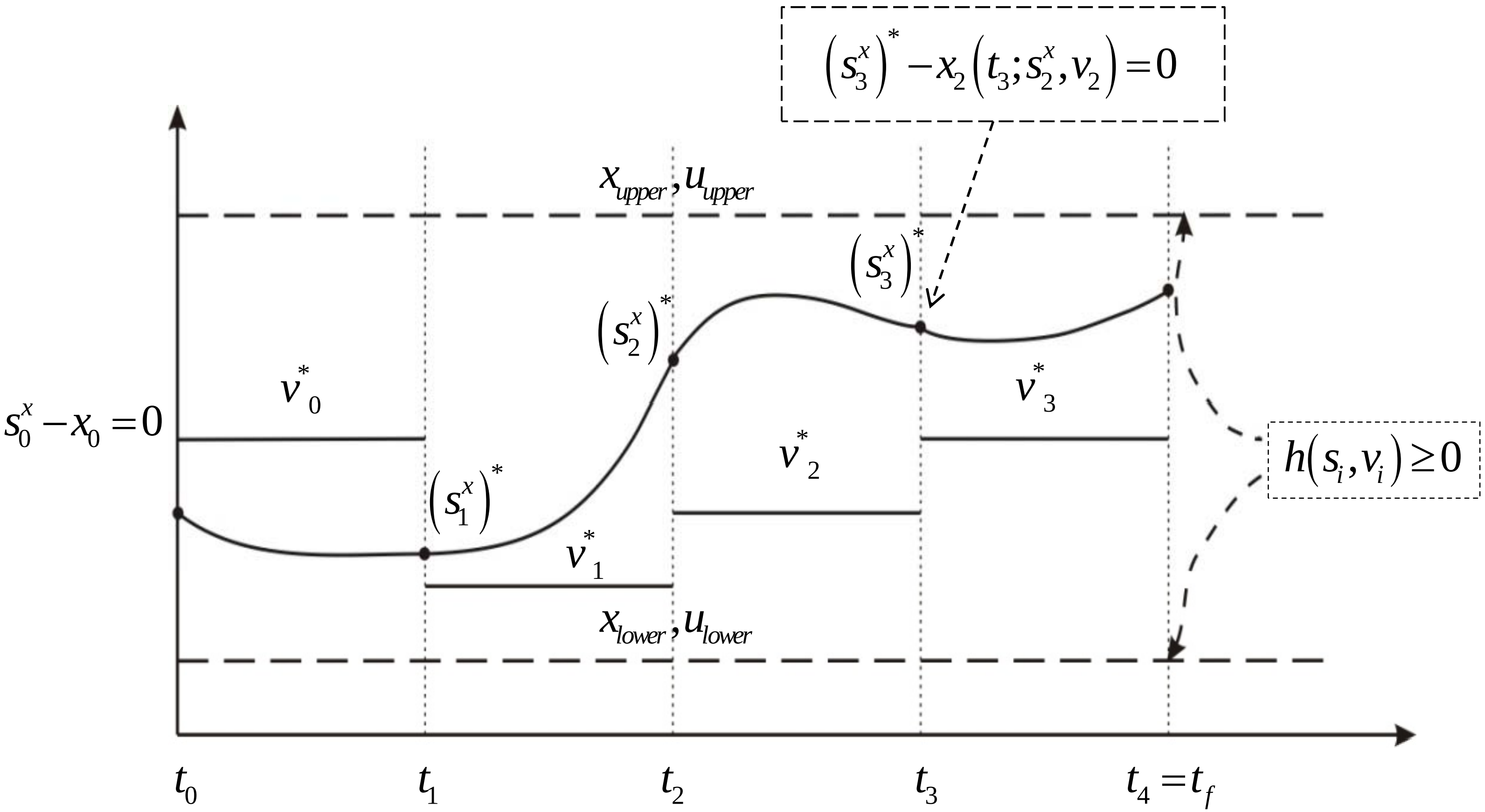
$$0 = r_e\left(s_N, p\right)$$

$$0 \le r_i\left(s_N, p\right)$$

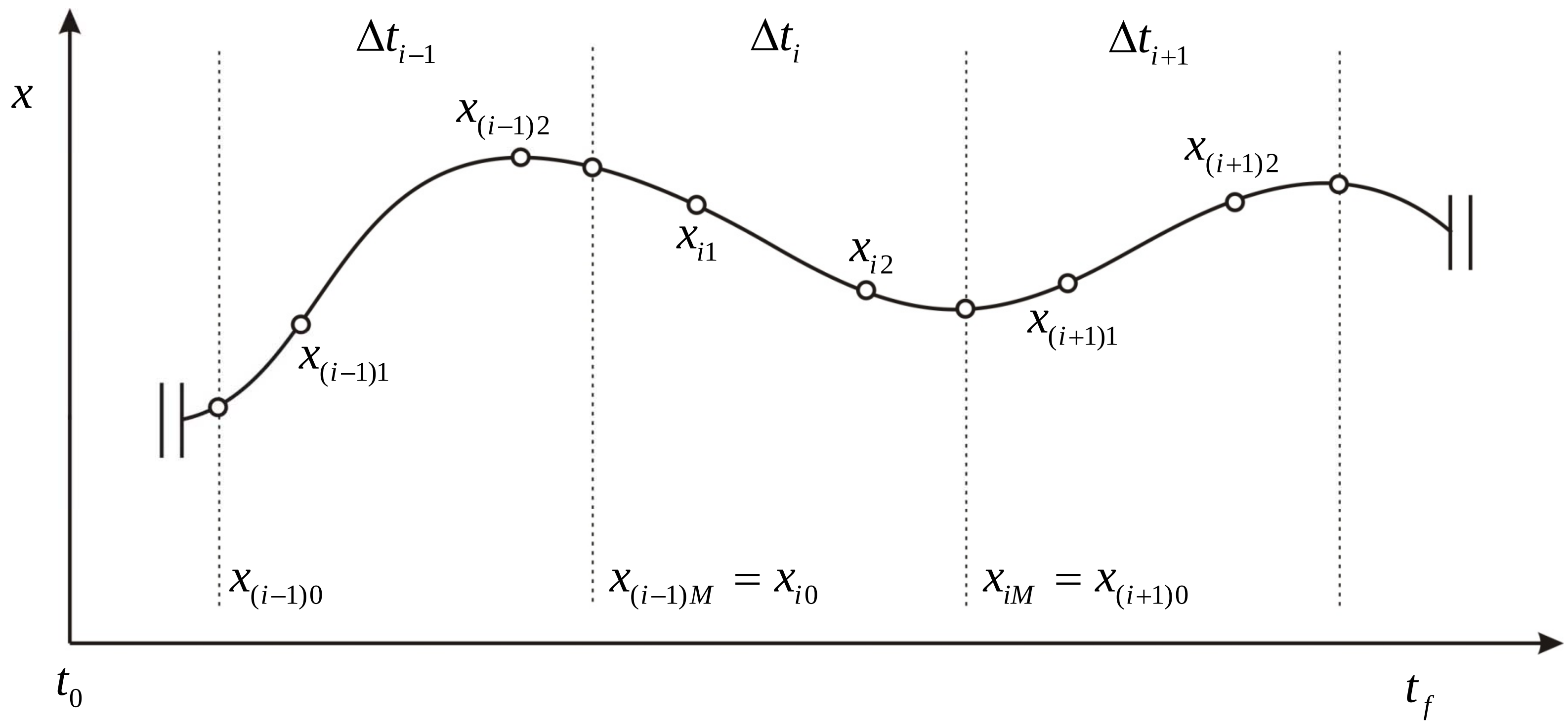
# Multiple shooting



# Multiple shooting - problem solution



# Collocation on finite elements



# Collocation on finite elements

$$x(t) = \sum_{k=0}^{K} x_{ik} l_k(\tau)$$

$$\frac{d}{dt} \qquad \frac{d}{d\tau}\frac{d\tau}{dt}$$

$$\dot{x}(t) = \frac{1}{h_i} \sum_{k=0}^{K} x_{ik} \frac{l_k(\tau)}{d\tau}$$

$$l_k(\tau) = \prod_{k \neq j=0}^{K} \frac{\tau - \tau_j}{\tau_k - \tau_j}$$

$$t \in [t_i, t_{i+1}]$$

$$t = t_i + h_i \tau$$

$$\dot{x}(t) = f(x,t)$$

$$\sum_{k=0}^{K} x_{ik} \frac{dl_k(\tau)}{d\tau} = h_i f(x,t)$$

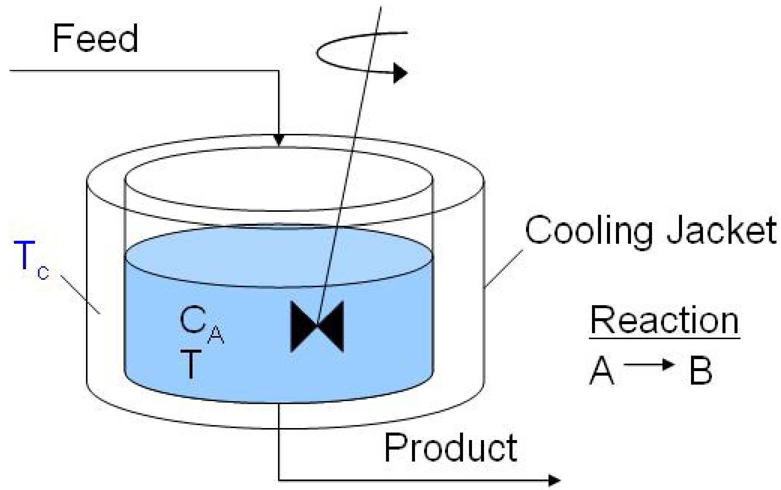
# Advantages of collocation on finite elements

- **Single-step method** ⇨ smooth profiles required only *within* the finite elements, discontinuity of control profiles at $t_i, i = 1, \ldots, N$ possible
- **High-order implicit method** ⇨ provides accurate profiles with relatively few finite elements, i.e. reduces problem size
- No stability limitations on element size for stiff systems
- Path constraints can be held inside the elements

# Case study - CSTR

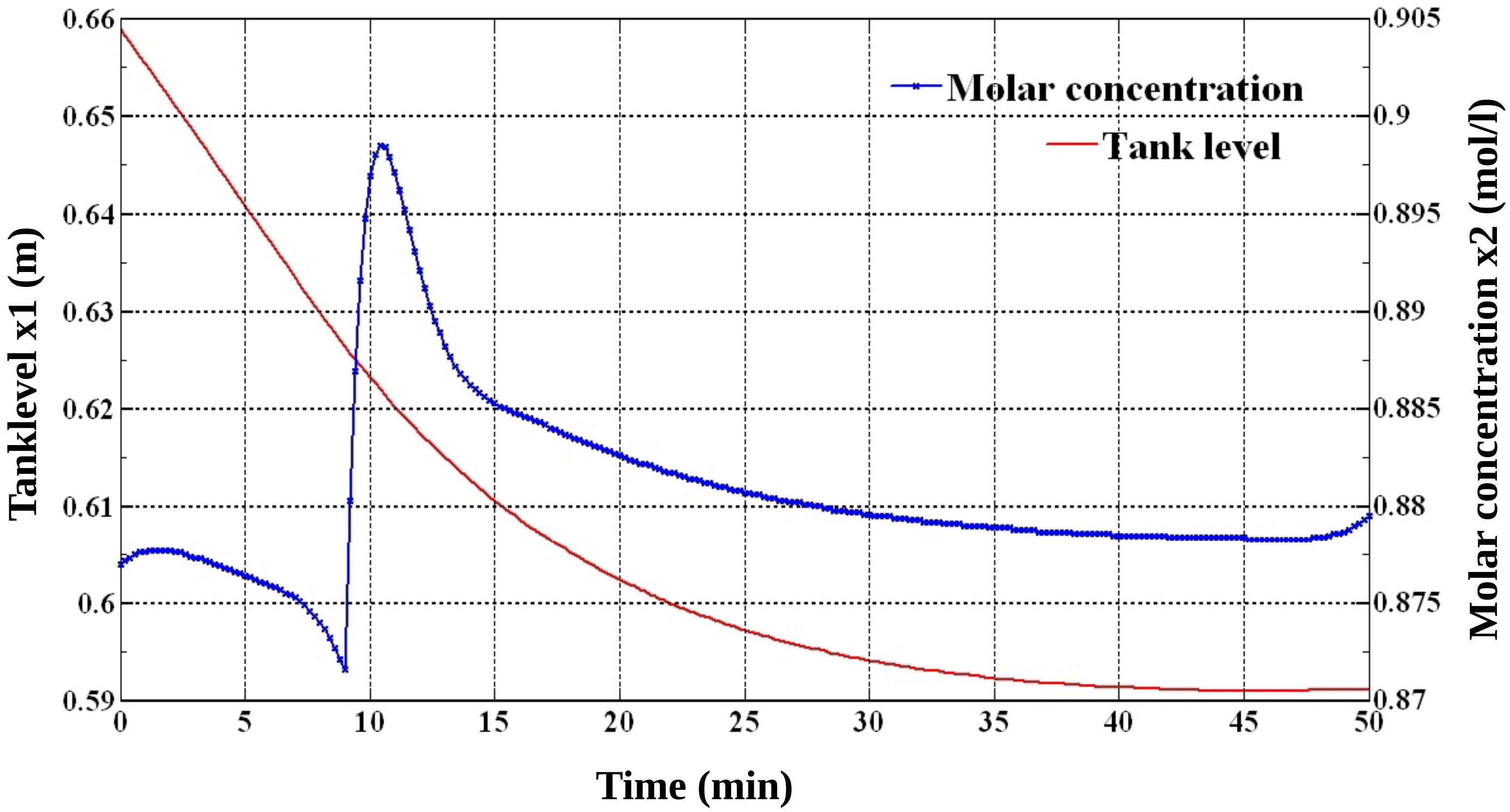
$$\min_{x,u} \int_{t_0}^{t_f} \sum_{i=1}^{N_x} w_i (x_i - x_i^{(ref)})^2 + \sum_{i=1}^{N_u} w_i (u_i - u_i^{(ref)})^2 dt$$

s.t. model equations
variable bounds

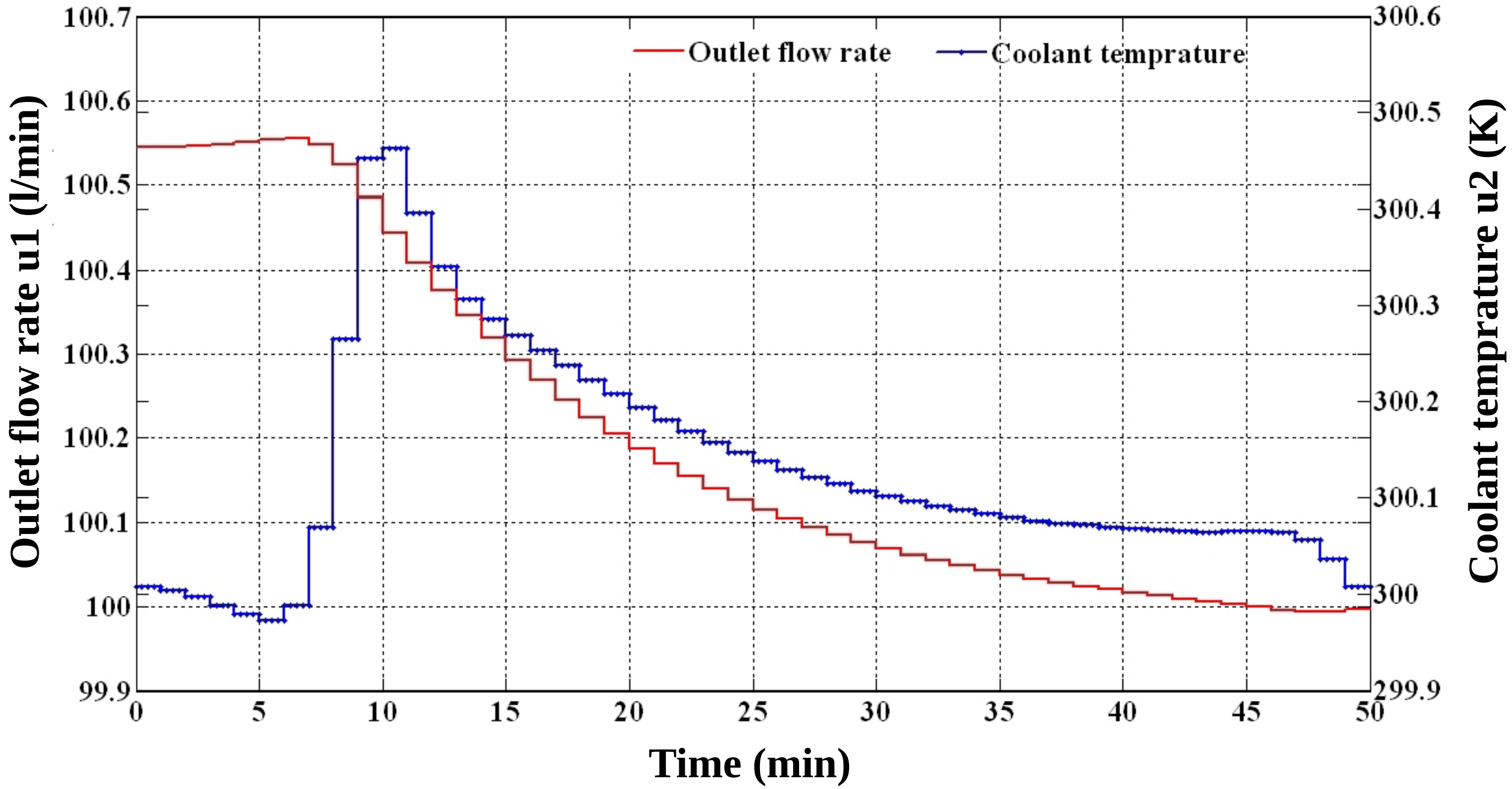



Using 50 subintervals,
IPOPT 3.4.0 optimizer.
CPU time is 0.953 sec.

# Case study - CSTR



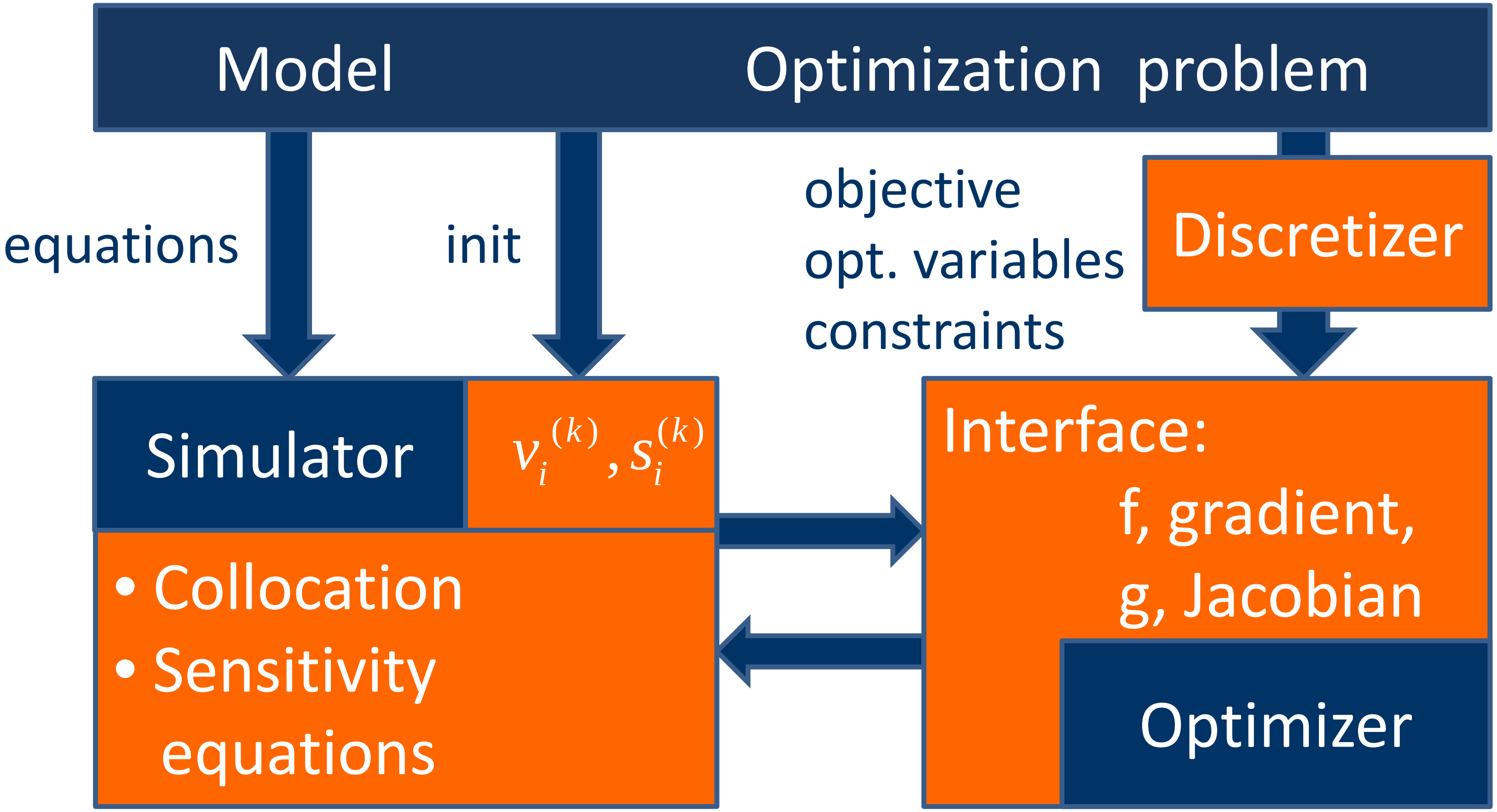
# Case study - CSTR



# Classification of methods – consequences for the optimization framework

| | Single shooting | Multiple shooting | Simultaneous approach |
|---|---|---|---|
| Optimization variables | $v_i$ <br> $i = 0, \ldots, N-1$ | $v_i, s_i$ <br> $i = 0, \ldots, N-1$ | $v_i, x_i$ <br> $i = 0, \ldots, N \times K$ |
| Problem size | small | intermediate | large |
| Simulation | 1 IVP | N IVP`s | None |
| Path constraints | none | fulfilled at $t_i, i = 0, \ldots, N \times K$ | fulfilled at $t_i, i = 0, \ldots, N \times K$ |

# Implementation in OpenModelica
# What is needed?



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# Summary

Collocation method: fast accurate simulator

Multiple shoothing: reduced problem size, path constraints incorporated

Multiple shooting with collocation: high accuracy and reduced computational effort

Structure of the algorithm: reveals what is needed for implementation in OpenModelica

# Many thanks for your attention!

# References

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[3] L. T. Biegler, Solution of dynamic optimization problem by successive quadratic programming and orthogonal collocation, Computers and Chemical Engineering 8 (1984), 243-248.
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